Induction

I Am ______________________

For years I used Proof By Induction, but never really understood why it worked. This frustrated me, and so I set out to discover the "proof" for proof by induction. The method we'll use today comes from Foerster's Precalculus book, where sadly the entire topic is relegated to an appendix.

Postulate: The Well Ordering Principle - If you have a non-empty set of positive integers then that set will have a least element.

Recall the formula for the sum of the first n natural numbers: $1 + 2 + ... + n =$

We will prove this formula is true using the Indirect Method, or Proof By Contradiction, you should remember this strategy from Geometry but I realize that was a long time ago...

Step 1: Assume Not - We are going to assume that there is some number k for which the statement is not true... Hence:

It is our job now to contradict this statement.
Alas, we know the statement is true if $n = 1$, let's show this now by plugging in $n = 1$ into both sides of our formula:

So the statement we are trying to prove is either true for a number or false for a number, this is an either or situation. We can put these situations into two boxes.

Numbers for which the property is TRUE	Numbers for which the property is FALSE

Let's populate these boxes, and give rationales for why each item goes in the chosen box...

So at this point we know the property is true for $n = l - 1$. Let's write this statement out...

Okay so let's reconsider the situation for $n = l$.

This is a contradiction. Which means that our Assume Not statement was false and our statement is indeed true for all the natural numbers. A proof by induction shortens this process into 3 easy steps.

Step 1: Prove the statement is true for $n = 1$ or some other base case
Step 2: Assume that the statement is true for $n = k$.
Step 3: Use 1 and 2 to Prove the statement is true for $n = k + 1$.

The steps are easy but the proofs, surprise, surprise, can get tricky. Let's do some...

Problem Set

1. Prove: $2 + 4 + 6 + ... + 2n = n^2 + n$ for all positive integers n.

2. Prove: $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$ for all positive integers.

3. Prove the formula for the sum of the first n terms of a geometric sequence: $S_n = \frac{a_1(1 - r^n)}{(1 - r)}$

   recall that a_1 is the first term and $r, r \neq 1$ is the common ratio.

4. Find a formula for the series, and then prove it by induction: $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + ... + \frac{1}{n \cdot (n+1)}$

5. Oh divisibility, prove that $9^n - 1$ is divisible by 8 for all n≥1 where n is an integer.

6. Prove that $3^{2n} - 1$ is a multiple of 8, for integers $n \geq 1$

7. Using mathematical induction prove that $\frac{d^n}{dx^n}(\cos x) = \cos\left(x + \frac{n\pi}{2}\right)$ for all positive integers.